J. Am. Chem. Soc. 1992, 114, 1606-1616

Simulation of MC-SCF Results on Covalent Organic Multi-Bond Reactions: Molecular Mechanics with Valence Bond (MM-VB)

Fernando Bernardi,[*,†] **Massimo Olivucci,**[§] **and Michael A. Robb**[*,§]

Contribution from the Dipartimento di Chimica "G. Ciamician" dell'Università di Bologna, Via Selmi 2, 40126 Bologna, Italy, and the Department of Chemistry, King's College, London, Strand, London WC2R 2LS, U.K. *Received June 3, 1991*

Abstract: A scheme, MM-VB, for modelling (i.e., simulating MC-SCF results) covalent multi-bond reactivity problems (pericyclic and radical reactions) using a combination of molecular mechanics (MM) and valence bond (VB) theory is presented. The efficiency of the model in locating equilibrium geometries, transition structures, reaction intermediates, excited-state structures, and structures associated with conical intersections of ground and excited states is documented by comparison with ab initio MC-SCF results for structures belonging to four different potential energy surfaces for hydrocarbon reactions. The results have been obtained using the same pool of parameters (71 parameters) for the valence bond parametrization in all computations. The method performs very well for the semiquantitative prediction of structural parameters, and it is always possible to deduce the correct shape (topology) of the global potential energy surface. The application of the method to larger molecules is illustrated with an example where the conformational change in the ring system of ergosterol during electrocyclic ring opening is discussed. From a conceptual point of view, it has been demonstrated that quantum mechanics of the complicated process of the reorganization of the spin recoupling that occurs in a transition state or the driving force for the change in CC bond lengths that occurs in delocalized π systems can both be described by the solution of a simple VB problem which allows for the *resonance* of the possible VB structures *involving the active bonds alone.* The inactive *framework* of the molecule can be described by MM. The central parameters in such a VB calculation are the exchange integrals (K_{ab}) and Coulomb integrals (Q_{ab}). The functional form of these Coulomb and exchange parameters can be transferred from two active-center model molecules (in the present case ethane and ethylene) to a multi-active-center system via a *delocalization* algorithm.

Introduction

With modern computers and standard software, it is now possible to make quite detailed investigations of organic reaction mechanisms (reaction intermediates, transition structures, and reaction paths) using ab initio methods. However, because of the cost, such investigations will always be limited to prototype examples of certain classes of reaction. In contrast, for structural problems, molecular mechanics (MM) provides an everyday laboratory tool that can be used before detailed experimental investigations are carried out. The difficulty with chemical reactivity problems is that the complicated electronic reorganizations that accompany bond breaking/making require a quantum mechanical description. The purpose of this paper is to demonstrate that it is possible to model even the quantum mechanics using the valence bond (VB) method in the form of a parametrized Heisenberg Hamiltonian. In this initial implementation, we shall limit ourselves to intermediates and transition states that involve making/breaking one σ/π bond at any carbon atom. Thus we shall be concerned with the making/breaking of *covalent* bonds only, and in this first study we do not allow for the case where a pair of electrons is transferred in the formation of a *dative* bond or for the case of an ionic reaction such as an S_N2 which involves both charge transfer and bond exchange.

As we shall show, even though our model is still in an early stage of development compared to the current sophisticated status of MM methods, a parametrized VB model when combined with a standard MM force field can locate, with reasonable accuracy, the structures of minima, transition structures, and reaction intermediates in prototypical multi-bond for ground- and excited-state reactions using the same pool of VB parameters. Furthermore, the structures generated in MM-VB provide starting geometries for ab initio MC-SCF calculations that are sufficiently accurate that rapid convergence in the geometry optimization is usually achieved. This opens the possibility of exploiting the technique as a guideline for more accurate but still expensive ab initio calculations by providing a pool of approximate critical point structures and a knowledge of the topology of the potential energy surface for the reaction under investigation. (By topology, we mean the number and nature of intermediates/minima/transition states and possible reaction paths.) Armed with such knowledge, ab initio computations, particularly on larger systems, can be carried out very efficiently.

The molecular mechanics (MM) method (see ref 1-4 for general reviews) has evolved in many forms to become a standard tool for the study of *structural* problems. The information provided by MM is only as reliable as one's initial chemical skill in choosing the local environment defined by different atom types for the problem at hand. MM cannot decide upon or adjust the local environment itself. Of course, one could define a new local environment (and a set of parameters) for a transition state, and Houk and co-workers[5] have described such an approach. However, because the different atom types are defined from the outset, one cannot describe the complete potential energy surface for a reaction where the atom type changes in the course of reaction. In order to model chemical reactivity in a completely general way, one must combine a MM model with some sort of model that has its origins in the quantum mechanics that is necessary to describe the electronic reorganization that occurs during a chemical reaction. Warshel and co-workers[6] have used this approach successfully in a number of ways. In general, two strategies are possible: one must either combine rigorous ab initio quantum theory with MM or parametrize the quantum theory in the same manner that one parametrized MM itself. In this paper we shall show that a method based upon the simulation of MC-SCF results, using the VB method, yields results of an acceptable accuracy yet adds only a small overhead to the cost of an MM calculation itself. While the ability to model reactivity with MM and quantum theory is

---

[†] Dipartimento Chimico "G. Ciamician" dell'Universita di Bologna.

[§] Chemistry Department, King's College, London.

(1) Boyd, D.; Lipkowitz, K. *J. Chem. Educ.* **1982**, *59*, 269.

(2) Allinger, N. L. *Adv. Phys. Org. Chem.* **1976**, *13*, 1.

(3) Allinger, N. L.; Yuh, Y. *QCPE* **1980**, *12*, 395. Burkert, U.; Allinger, N. L. *Molecular Mechanics* ACS Monograph Series, No. 177; American Chemical Society: Washington, DC, 1982.

(4) Clark, T. *Handbook of Computational Chemistry*; Wiley: New York, 1985.

(5) Spellmeyer, D. C.; Houk, K. N. *J. Org. Chem.* **1987**, *52*, 959.

(6) (a) Warshel, A.; Lappicirella, A. *J. Am. Chem. Soc.* **1981**, *103*, 4664. (b) Warshel, A.; Weiss, R. M. *J. Am. Chem. Soc.* **1980**, *102*, 6218.

0002-7863/92/1514-1606$03.00/0

of great practical importance, the fact that one can parametrize a simple model derived from quantum mechanics in this fashion is important as well because it demonstrates an underlying simplicity in the bond making/breaking process.

Thus our purpose in this paper is to describe our implementations of a molecular mechanics valence bond method (MM-VB) in which the bond making/breaking process is described using VB theory, and the remainder of the interactions are described by MM. The basic groundwork for this approach was laid in the work of Malrieu, Durand, Daudey, and their co-workers[7] who first demonstrated that the VB method can be successfully parametrized for conjugated hydrocarbons from an ab initio computation using effective Hamiltonian theory. In other work,[8-11] we have shown how their approach can be extended to model chemical reactivity using a VB model with very simple potentials for the framework of the molecule. In this paper we describe a VB parametrization of the bond breaking/making process that is quite general yet has the particular feature that it is designed to be combined with the *standard* MM2[2] parametrization of the molecular framework. Because our VB parametrization can describe the behavior of the VB parameters as a function of sp^2 to sp^3 hybridization (i.e., the pyramidalization angle), and because we allow for atomic orbital distortion via a delocalization algorithm, the method can be used to study reactivity problems of hydrocarbons.

MM-VB for a Single *Active* Covalent Bond

A VB formulation of the bond breaking/making process is attractive because as chemists we believe that we can attach physical significance to the various VB covalent configurations. However, we would like to emphasize another aspect. VB wave functions can always be written as eigenfunctions of what is known as a Heisenberg spin Hamiltonian (see ref 7 for a full discussion of the background to this approach). The parameters of this Hamiltonian have a simple physical interpretation, and this provides a very accurate way of *simulating*[10] ab initio results. We shall illustrate all the main ideas with a simple two-electron two-orbital model of a single bond.

We begin with H_2. A covalent VB wave function for a single bond in H_2 can be written as a combination of two determinants:

$$|\Phi_a\bar{\Phi}_b| \tag{1a}$$

and

$$|\Phi_b\bar{\Phi}_a| \tag{1b}$$

where Φ_a and Φ_b are atomic orbitals. The Heisenberg spin Hamiltonian is defined in terms of its matrix elements (Q_{ab} and K_{ab}) in the basis of covalent determinants as

$$\mathbf{H}_s = \begin{bmatrix} Q_{ab} & K_{ab} \\ K_{ab} & Q_{ab} \end{bmatrix} \tag{2}$$

In a minimum AO basis for H_2, the parameters Q and K are the usual Coulomb and exchange integrals

$$(1 + \langle\Phi_a|\Phi_b\rangle^2)Q_{ab} = \langle\Phi_a|\hat{\mathbf{h}}|\Phi_a\rangle + \langle\Phi_b|\hat{\mathbf{h}}|\Phi_b\rangle + \left[\Phi_a\Phi_a\left|\frac{1}{r_{12}}\right|\Phi_b\Phi_b\right] + V_{NN} \tag{3a}$$

$$(1 + \langle\Phi_a|\Phi_b\rangle^2)K_{ab} = \langle\Phi_a|\Phi_b\rangle\{\langle\Phi_a|2\hat{\mathbf{h}} + V_{NN}|\Phi_b\rangle\} + \left[\Phi_a\Phi_b\left|\frac{1}{r_{12}}\right|\Phi_a\Phi_b\right] \tag{3b}$$

where $\mathbf{h}$ is the one-electron kinetic + electron attraction operator, V_{NN} is the nuclear–nuclear repulsion, and r_{12}^{-1} is electron–electron repulsion. The solution of the usual secular equation involving the spin Hamiltonian in eq 2 yields (apart from normalization) the singlet state

$$\Psi = |\Phi_a\bar{\Phi}_b| - |\bar{\Phi}_a\Phi_b| \tag{4}$$

as the lowest energy solution with energy

$$E = Q_{ab} + K_{ab} \tag{5}$$

The preceding discussion would appear to be restricted to a minimum basis of atomic orbitals and also appears to omit the important *ionic* VB configurations. However, this is not the case at all! Provided the singlet and triplet energies E^1 and E^3 are available from an ab initio computation we have

$$\begin{matrix} Q_{ab} \\ K_{ab} \end{matrix} = \frac{(E^1 \pm E^3)}{2} \tag{6}$$

and it is clear that one could fit the singlet and triplet energies as a function of distance with two parameters Q_{ab} and K_{ab}. Consequently, one can obtain *numerical* values of the energy, and of the parameters Q_{ab} and K_{ab} to any desired degree of accuracy, limited only by the AO basis and the accuracy of any quantum mechanical method used. One is then free to rationalize the behavior of Q_{ab} and K_{ab} in terms of the undefined nonorthogonal orbitals in eq 3a and 3b. Alternatively, any CI wave function can be projected onto the space defined by the two VB determinants (1a)/(1b) and in this case H_s (eq 2) becomes an effective Hamiltonian[7,10] that reproduces the CI energy *exactly*. (The effective Hamiltonian approach has the advantage that it can be used for more than two electrons and is compatible with the discussion that follows in the multi-bond problem later in this article.) Thus rather than explicitly evaluating Q_{ab} and K_{ab} via eq 3a/3b using a minimum basis, the AO basis becomes *implicit* and the Q_{ab} and the K_{ab} are regarded as *parameters* in the effective Hamiltonian which has been projected from a large-scale CI in an extended basis. We have given a pedagogical discussion of the nature of the implicit AO basis elsewhere.[10b] It is sufficient to remark that the *implicit* AO basis is distorted so that the exact energy can be simulated using the covalent terms alone and the ionic terms become redundant.

The utility of the strategy described above arises because both Q_{ab} and K_{ab} have a simple behavior as a function of the interatomic distance r_{ab}. As a consequence one can fit accurately computed Q_{ab} and K_{ab} data using a rather simple functional form. The result is an analytical expression for the *simulated* matrix elements of the ab initio VB Hamiltonian. For example, even simplest exponential functions of the form

$$K_{ab}(r_{ab}) = A\exp(-Br_{ab})$$

$$Q_{ab}(r_{ab}) = C\exp(-Dr_{ab}) - E\exp(-Fr_{ab})$$

can give a reasonable approximation to Q_{ab} and K_{ab} for the H_2 case.

To summarize, by fitting the matrix elements Q_{ab} and K_{ab} as a function of r_{ab} and using the definition of the Heisenberg Hamiltonian in eq 2, one has an analytical model for potential energy curves of the lowest singlet and triplet states of H_2. The two parameters $Q_{ab}(r_{ab})$ and $K_{ab}(r_{ab})$ can be obtained to any desired degree of accuracy using effective Hamiltonian methods with a CI or MC-SCF wave function.

Now let us discuss the description of a particular C–C covalent bond in a general molecule. The preceding example can be easily generalized. One needs only to define the concept of *active* and *inactive* orbitals. The *active* orbitals play the role of Φ_a and Φ_b in the preceding discussion and are the orbitals involved in bond breaking/making between atomic sites a and b. The remainder

---

(7) Said, M.; Maynau, D.; Malrieu, J. P.; Bach, M. A. G. *J. Am. Chem. Soc.* **1984**, *106*, 571. Maynau, D.; Durand, Ph.; Duadey, J. P.; Malrieu, J. P. *Phys. Rev. A* **1983**, *28*, 3193. Durand, Ph.; Malrieu, J. P. *Adv. Chem. Phys.* **1987**, *68*, 931.

(8) Bernardi, F.; De, S.; Olivucci, M.; Robb, M. A. *J. Am. Chem. Soc.* **1990**, *112*, 1737-1743.

(9) (a) Bernardi, F.; Olivucci, M.; Robb, M. A. *Acc. Chem. Res.* **1990**, *112*, 405-412. (b) Bernardi, F.; Olivucci, M.; Robb, M. A. *Res. Chem. Intermed.* **1989**, 217-249.

(10) (a) Bernardi, F.; Olivucci, M.; McDouall, J. J. W.; Robb, M. A. *J. Chem. Phys.* **1988**, *89*, 6365. (b) Robb, M. A.; Bernardi, F. *New Theoretical Concepts for Understanding Organic Reactions*; Bertran, J., Csizmadia, I. G., Eds.; Kluwer Academic Publishers: Dordrecht, 1989; pp 101-146.

(11) Bearpark, M.; Bernardi, F.; Olivucci, M.; Robb, M. A. *J. Am. Chem. Soc.* **1990**, *112*, 1732-1736.

of the orbitals are *inactive*. While the parameter K_{ab} has the same physical interpretation as in the preceding discussion, the Coulomb term must be modified to incorporate the framework ($Q_{framework}$) of the molecule that contains the inactive orbitals. Thus Q_{ab} in eq 2 is replaced by Q which is defined as

$$Q = Q_{framework} + Q_{ab}$$

The parameter Q contains the energy of the *framework* of the molecule, $Q_{framework}$, in addition to the parameter Q_{ab} which contains the terms involving the active orbitals implied by eq 3a. Again, using effective Hamiltonian methods one can obtain the *numerical* values of the parameters Q and K_{ab} to any desired degree of accuracy.

While the computation of the *numerical* values of Q and K_{ab} for a given interatomic distance r_{ab} can be accomplished in the same way as in the H_2 example discussed above, the task of *parametrizing* the effective Hamiltonian is slightly more difficult because (a) the functional form of the representation of Q_{ab} and K_{ab} must depend on the orientation and on the hybridization of the C atom active orbitals (sp^n hybrids), and (b) the parameter Q contains the energy, $Q_{framework}$, of the *framework* of the molecule. Thus orientation and the hybridization changes for an sp^3–sp^2 carbon atom must be incorporated into the functional form used to fit the data. We have used a functional of the form

$$Q_{ab} \text{ or } K_{ab} = \sum_k^n C_k(a_k \exp\{-b_k r_{ab}\}) \qquad (7)$$

where the coefficients C_k describe orientation and the hybridization changes as described in the Appendix. For K_{ab} we have used 4 exponentials (i.e., $n = 4$ and 8 parameters) while for Q_{ab} 11 exponentials are required ($n = 11$ and 22 parameters). Parametrizing that part of Q that contains the energy of the *framework*, $Q_{framework}$, of the molecule would be a difficult task in general. For this part of Q we shall use the standard MM potential for terms that contain the atoms that do not involve active orbitals, and we shall use a modified MM potential for terms (i.e., stretching, bending, and torsion) that contains atoms that have one or more active orbitals. Thus, the central feature of this work is the use of a potential for Q of the form

$$Q = V_{MM} + Q_{ab} \qquad (8)$$

where V_{MM} is the standard MM potential for the atoms that do not involve active orbitals and a modified MM potential for atoms that have an active orbital. Q_{ab} and K_{ab} are to be parametrized in the same way as in the preceding example. The details of the resulting function are given in the Appendix.

A large group of hydrocarbon reactions are characterized by bond-making and bond-breaking processes involving carbon–carbon σ and π bonds. The reactive systems belonging to this group can be visualized as if they were composed of isolated pairs of two-active-carbon subsystems involving a σ or a π bond which undergoes large geometrical and hybridization changes as during bond breaking/making. Thus the first step in modelling the potential energy of this group of reactions is achieved by modelling the potential surface of *model* two active-carbon subsystems. The interaction of these isolated two active-carbon subsystems is then achieved by allowing for spin recoupling via *resonance*. Thus the second step involves the computation of such resonance effects which we will discuss in the next subsection.

The potential energy surfaces of the ethane and ethylene molecules provide the desired two active-center *model* molecules for the isolated σ and π bond of a reacting hydrocarbon. In ethane the active orbitals are the carbon sp^3 orbitals involved in the σ bond. We aim for the parametrization of Q and K_{ab} that can describe ethane, two methyl radicals (where the C atoms are sp^2), and any intermediate situation characterized by an intermediate C–C bond length and hybridization status. In ethylene, the active orbitals are the sp^2 carbon p$^\pi$ atomic orbitals. Here we aim for a parametrization that can describe ethylene itself (including rotation about the σ bond) through to the limiting situation where the carbon p$^\pi$ atomic orbitals are sp^3. This is accomplished by requiring the C_k, a_k, and b_k in eq 7 to be functions of the hybridization status and relative orientation of the C atom active orbitals. In general, the standard parametrization of V_{MM} is used to describe the framework of the system. *There are no MM parameters that refer to forming bonds between atoms that have active orbitals with the exception of the torsional parameters about forming the C–C σ bond that must now be functions of* the *magnitude of the interaction* (K_{ab}) *between* the *C atoms with active orbitals*. Thus the bending potential that involves a forming bond arises naturally in the VB problem. The only modification to the standard MM potential involves the bending potential that contains an *anchor* atom with an active orbital. This bending potential now takes on a functional dependence on the hybridization status of the active orbital that reduces to the standard MM potential for sp^2 hybridization. The details can be found in the Appendix.

Potential functions of the form just described were parametrized for ethane and ethylene using effective Hamiltonians obtained from ab initio MC-SCF computations in a 4-31G basis using techniques described in detail elsewhere.[10,11] As we have discussed earlier, the Q_{ab} and K_{ab} can be determined to any desired level of accuracy depending on the AO basis and quantum mechanical procedure used. Thus there would be no difficulty in including dynamic correlation and much larger basis sets. Since, for the simulations to be reported in the second part of the paper, we had detailed results on several reactivity problems at the MC-SCF/4-31G level, the current choice of method/basis set is appropriate. In any case, in this initial implementation, we were concerned primarily with reproducing surface topology, and for this task the quantum mechanical technique used is adequate.

The parameters were obtained via a standard least-squares procedure applied to the data from MC-SCF results for 176 geometries (128 for ethane and 48 for ethylene). The geometries were chosen so as to sample the behavior of the Q_{ab} and K_{ab} as a function of the most important framework deformations that occur during bond breaking for the two model systems. Thus the bond breaking (C–C bond stretch) was parametrized for both ethane and ethylene including the effect of hybridization status via variable pyramidalization (see Figure 1a for ethane) of the two CH_3's in ethane and the two CH_2's in ethylene, including the effect of the twisting angle for the ethylene molecule (see Figure 1b) and including the effects of different orientations of the CH_3 group axes (relative to the C–C bond axis) for ethane. In addition, the pyramidalization energy of a single CH_3 radical was parametrized in order to simulate the behavior of a noninteracting CH_3 and $-CH_2$ radical centers.

The average difference in energy between the ab initio and parametrized potentials is 0.6 kcal/mol, but in certain regions of the surface the error was as large as 6.0 kcal/mol. The largest errors occur in those regions of the surface that are far away from the minimum energy valley for bond dissociation (e.g., geometries with quasi-planar CH_3 fragments in conjunction with short C–C bond distances in ethane). Since these regions of our potential are never encountered in the simulations we report in this paper, such errors are of no consequence. In general, the optimized C–C distances were in agreement with the ab initio data to within 0.03 Å. The rotation barrier in ethane has an error of 0.5 kcal/mol and in ethylene 3.8 kcal/mol. However, it is the global shape that is important. This can be demonstrated by plotting some potential surfaces. In Figure 1a we show an ab initio MC-SCF potential energy surface of ethane in the space of the r_{CC} distance (diagonal top right to bottom left, 15 gridnodes) and the CH_3 pyramidalization (diagonal bottom left to bottom right, 5 gridnodes). The σ bond dissociation and the change in the CH_3 pyramidalization from 120° to around 100° in this process cannot be described using MM alone. In Figure 1b we show the MC-SCF surface (lower) together with the MM-VB (upper) as contour diagrams to facilitate comparison. It can be seen that MM-VB correctly allows σ bond dissociation. In Figure 1, c and d, we show similar potential energy surfaces of ethylene as a function of r_{CC} (diagonal top right to bottom left, 6 gridnodes) and the torsional angle about CC (diagonal top left to bottom right, 11 gridnodes). The important feature of Figure 1c is the breaking of the π bond by rotation about

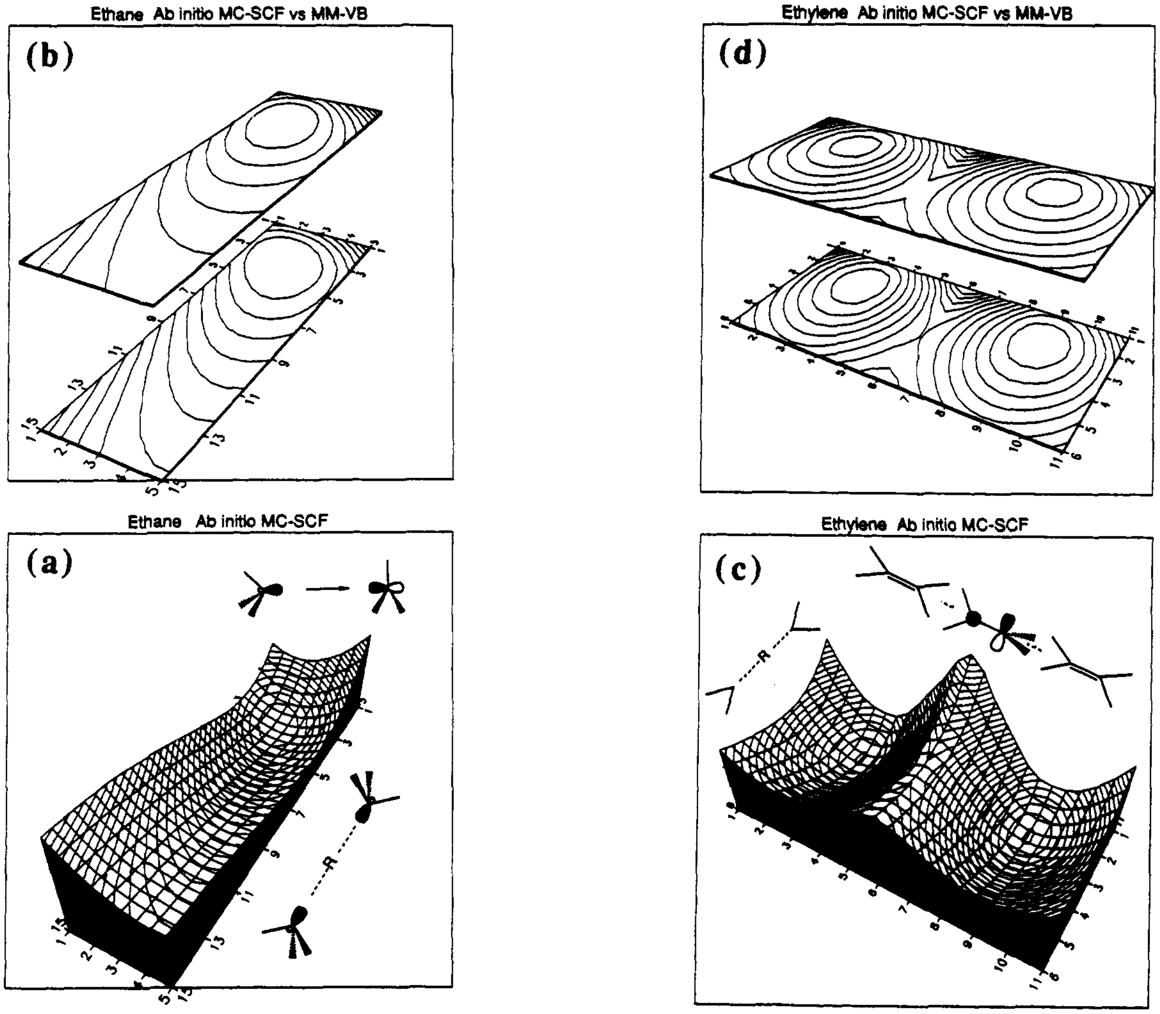
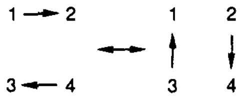


Figure 1. Comparison of MM-VB and 4-31G MC-SCF potential energy surfaces. a and b correspond to the fragmentation (Y axis, diagonal top right to bottom left) of ethane into two methyl radicals coupled with CH_3 pyramidalization (X axis, diagonal top left to bottom right). (a) MM-VB surface, (b) comparison of MC-SCF surface (top slice) and MM-VB (bottom slice). There are 15 gridnodes on the Y axis, the first corresponding to an r_{cc} distance of 1.34 Å and the last to 2.7 Å. There are 5 gridnodes on the X axis, the first corresponding to a HCC angle of 119.4°, the last to 99.4°. c and d correspond to the C–C stretch (Y axis, diagonal top right to bottom left) of the ethylene double bond coupled with terminal methylene rotation (X axis, diagonal top left to bottom right). (a) MM-VB surface, (b) comparison of MC-SCF surface (top slice) and MM-VB (bottom slice). There are 5 gridnodes on the Y axis, the first corresponding to an r_{cc} distance of 1.2 Å and the last to 1.7 Å. There are 11 gridnodes on the X axis, the first corresponding to a HCCH angle of 60°, the last to −60°.

the double bond. From the comparison of the contour diagrams in Figure 1d we see that this process is correctly described using MM-VB. From these data, one can conclude that functional form for the Q and K_{ab} is sufficiently flexible to describe both the σ/π bond formation and sp^2 to sp^3 hybridization changes.

MM-VB for Multi-Bond Reactivity Problems

We now consider the general problem of multiple covalent bonds. The Heisenberg Hamiltonian has the same general form; one simply has more covalent determinants. We can illustrate it with an example using four orbitals and four electrons. For a singlet state there are two singlet VB structures[12,13]



I II

These two singlet structures are combinations of six covalent determinants.

(1) $|3\ 4\ \bar{1}\ \bar{2}|$ (2) $|2\ 4\ \bar{1}\ \bar{3}|$ (3) $|2\ 3\ \bar{1}\ \bar{4}|$

(4) $|1\ 4\ \bar{2}\ \bar{3}|$ (5) $|1\ 3\ \bar{2}\ \bar{4}|$ (6) $|1\ 2\ \bar{3}\ \bar{4}|$

And the Heisenberg Hamiltonian $\mathbf{H}_s$ has the form

$$\begin{array}{ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & Q-K_{12}-K_{34} & K_{23} & -K_{24} & -K_{13} & -K_{14} & \\
2 & & Q-K_{13}-K_{24} & K_{34} & K_{12} & & \\
3 & & & Q-K_{14}-K_{23} & & & \\
4 & & & & & & \\
5 & & & & & &
\end{array}$$

(Matrix elements between pairs of determinants that differ by two spin interchanges, i.e., 1,6 or 2,5 or 3,4, are equal and not shown. Thus $H_{12} = H_{56}$, $H_{15} = H_{63}$, etc.)

The general form of $\mathbf{H}_s$ can always be written as

$$\langle K|\mathbf{H}_s|L\rangle = \delta_{KL}Q + \sum_{ab}\Gamma_{ab}^{KL}K_{ab} \tag{9}$$

where K and L index the determinants and a and b run over the active orbitals. The $\Gamma_{ab}{}^{KL}$ are simply numerical coefficients (0,

---

(12) McWeeny, R.; Sutcliffe, B. *Methods of Quantum Mechanics*; Academic: New York, 1969.

(13) Eyring, H.; Walter, J.; Kimball, G. *Quantum Chemistry*; Wiley: New York, 1944.

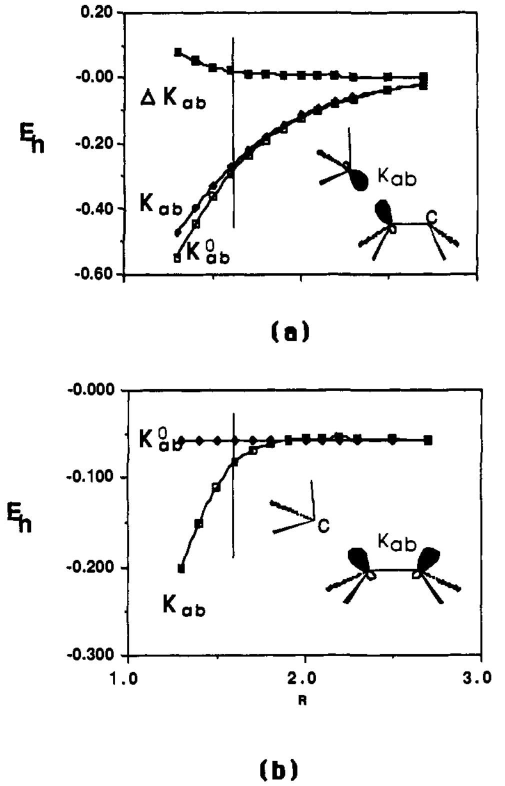


Figure 2. The delocalization effect (a) on a model σ bond and (b) on a model π bond. $K_{ab}{}^0$ is the exchange integral on the model system and K_{ab} the exchange integral in the *real* environment. The *X* axis is the distance (R) $CH_3^{\bullet}$ --- $CH_2{=}CH_2$ in Å. The results are obtained from MC-SCF computations at the 4-31G basis level.

±1) that can be computed from the usual rules for the evaluation of matrix elements over determinants.[12,13] The Coulomb energy Q is assumed to have the form

$$Q = V_{MM} + \sum_{ab} Q_{ab} \tag{10}$$

Thus given a parametrization of Q_{ab} and K_{ab} of the form discussed in the previous section, the generalization to an arbitrary number of electrons is immediate. The diagonalization of $\mathbf{H}_s$ gives us access to ground and excited states of any spin multiplicity.

We shall now complete the construction of a simulated potential for the group of hydrocarbon reactions discussed in the previous section. The new feature that must be addressed is that we must allow for the fact that Q_{ab} and K_{ab} in the true environment will not be the same as the Q^0_{ab} and K^0_{ab} of the two model molecules because the component two-active-center subsystems of the reacting hydrocarbon interact with each other. This *delocalization effect* is easily understood using modern VB theories[14] where the atomic orbitals Φ_a and Φ_b are now "raw" atomic orbitals but rather distort (or delocalize) in the molecular environment. Accordingly, the *delocalization effect* has its origins in the optimization of the AO basis (that is necessary to remove the ionic configurations) in modern spin coupled VB.[14]

We now give a simple example that illustrates the nature of the delocalization effect. Let us consider the surface (MC-SCF at the 4-31G basis level) for $CH_3^{\bullet}$ attacking a *frozen* (i.e., fixed geometry) $CH_2{=}CH_2$. In Figure 2 we show K^0_{ab} (calculated for the model systems), and the K_{ab} (calculated in the molecular environment) for the approach of $CH_3^{\bullet}$ to a *frozen* $CH_2{=}CH_2$. In Figure 2a we show the behavior of the K_{ab} associated with the forming σ bond which has been perturbed by the presence of the π bond of the *frozen* $CH_2{=}CH_2$. In contrast, in Figure 2b we show the behavior of the K_{ab} associated with the frozen π bond of $CH_2{=}CH_2$ which has been perturbed by the presence of the incoming $CH_3^{\bullet}$ which forms a σ bond. A vertical bar indicates the equilibrium CC bond distance associated with the σ bond in ethane for reference. For the forming σ bond, the difference (Figure 2a) between the environment-dependent K_{ab} and the reference K^0_{ab} is small except at small CC distances. On the other hand, the difference between the environment-dependent K_{ab} and reference K^0_{ab} (which is constant in this example because the $CH_2{=}CH_2$ fragment is frozen) for a π bond (Figure 2b) is very large. The differences between K^0_{ab} and K_{ab}, for both the σ bond and the π bond, are a function of the magnitude of K^0_{ac} (and thus the overlap $\langle\Phi_a|\Phi_c\rangle$) between the orbital centered on a or on b with the other active orbitals c in the molecule. In the true environment K_{ab} (Figure 2a) associated with the π bond grows larger than the reference K^0_{ab} as the $CH_3^{\bullet}$ fragment approaches because the ethylenic p^{π} orbitals delocalize toward the incoming $CH_3^{\bullet}$ fragment and their overlap increases. In the limiting case, when the bond is formed with a twisted ethylene in which the two p orbitals are orthogonal, the K_{ab} for the σ bond of the example discussed above (Figure 2a) is exactly equal to K^0_{ab}. The same is true for K_{ab} of the ethylenic π bond when the methyl radical is attacking in an orientation where its orbital cannot overlap with the ethylenic orbitals.

It would be very difficult to "model" this delocalization effect in general. However, our task is much simpler: we need only model the difference in the delocalization effect between the model environment and the real one. In order to "simulate" it, one has no alternative but to examine a large number of ab initio computations to attempt to discover some general rules. As a result of this process, we have a model which works reasonably well. However, our ability to model this effect is the factor which limits the accuracy of the method. We have used a simple functional of the form

$$K_{ab} = K^0_{ab} + \Delta K_{ab} \tag{11a}$$

$$\Delta K_{ab} = f\Big(\sum_{c\neq a,b} K^0_{ac}, \sum_{c\neq a,b} K^0_{bc}\Big) \tag{11b}$$

In eq 11 one is assuming that a given K_{ab} is affected by all the K^0_{ac} where c is the index of an active orbital on another site. Thus, referring to our example discussed previously ($CH_3^{\bullet}$ + $CH_2{=}CH_2$), we assume that the π bond K_{ab} is modified by magnitude of the σ bond K^0_{ac}. The detailed form of the functional we have used which contains 11 parameters is given in the Appendix. Since Q_{ab} is affected by the same kind of effects as K_{ab}, we use a similar functional.

Energetics in MM-VB

The prediction of energetics in MM and MM-VB is fundamentally different. In MM the reference point is a "ball and spring" model where the energy zero is associated with the reference bond lengths and angles. Thus the MM energy consists of the strain of this "ball and spring" model counterbalanced by electrostatic and van der Waals forces. Thus the MM method cannot describe the energy difference between say a product formed between two reactants and the reactants themselves. Only the difference in strain energies is described; the remainder of the energy is due to the new bonding situations and cannot be described at all in MM. Rather, one must use thermochemical and/or theoretical data in conjunction with MM energies. In contrast, in MM-VB we attempt a much more ambitious quasi-absolute description of the energy in the VB part while retaining the MM model for part of the Coulomb energy. Because we can recover that part of the energy difference between reactants and products that is due to electronic reorganization and arises from the VB part of the method, one has a good prospect for getting the correct surface topology. However, in MM-VB one cannot accurately describe the electronic polarization due to the electronic rearrangement of the valence electrons that occurs in the framework of the molecule that lies outside of the active space.

In order to obtain accurate relative energetics (barrier heights) that will be comparable with experiment, one must use thermochemical and theoretical data in the same spirit as in MM itself.

---

(14) Cooper, D. L.; Gerratt, J.; Raimondi, M. *Adv. Chem. Phys.* **1987**, *67*, 319.

However, one is usually interested in differences in barrier heights that arise from substituent effects. So that absolute barrier heights are unimportant. However, we can expect that MM-VB will be more reliable than MM itself in discussing substitution effects. In MM one must make the assumption that differences in barrier heights that arise from substituent effects must be due to differences in strain energy. Thus in an MM approach to reactivity there can be no allowance for the situation where the mechanistic pathway actually changes upon substitution. However, we have shown elsewhere[9,11] that the topology of the potential surface can be changed completely (e.g., a synchronous mechanism becomes asynchronous) with subtle changes in Q, and it is a rather delicate balance between Coulomb and exchange that determines mechanistic preference. In MM-VB, because the energetics are accurate enough to reproduce the correct surface topology between reactants and products, we can allow for the change in mechanistic preference using substituents. Further, the correction of absolute energetics in MM-VB will now be more accurate than in MM itself since we can use theoretical data in a more sophisticated way. One can obtain optimum geometries and energies for model unsubstituted systems at a very high level of theory that includes large basis sets and electron correlation. The MM-VB calculations can then be run at these optimum geometries to obtain a correction factor for the energetics. Because the surface topology is already correct (because of the VB part of the method), the corrections are much smaller than in MM itself, and the assumptions made in correcting the energetics in this way will be much less severe.

Some Numerical Examples

From the outset we must emphasize that our objective is the prediction of reaction mechanisms for ground- and excited-state reactivity problems in organic reactivity. Our aim is to be able to locate transition structures accurately enough that the interesting mechanistic pathways can be identified before an expensive ab initio computation is used to refine the data. This is obviously a much more ambitious objective than in MM itself. If one were to use MM to study reactivity, then one would need to assume the existence of a transition state (an thus a mechanistic pathway) from the outset since one must define "transition state atoms" as the input to the MM procedure. In contrast in MM-VB we do not assume the existence of a specific mechanistic pathway. Thus the prerequisite condition for success is that MM-VB energetics must be accurate enough to reproduce the potential surface topology (i.e., relative positions of minima and transition states) in the region between reactants and products. Thus the topological accuracy of the method is, in this contest, much more important than the prediction of accurate barrier heights, etc., which can always be corrected using thermochemical or theoretical data as in MM itself. The situation is similar to ab initio computations where one assumes that the structural part of the problem (i.e., geometry optimization) can be solved at the SCF or MC-SCF level and one accepts that one must include electron correlation (e.g., via MP2 or CI) to get energetics that can be compared with experiment. Furthermore, from a practical point of view, MM-VB optimized geometries have been proved to be an excellent starting point for accurate but expensive ab initio calculation. Thus the MM-VB results provide a guideline for expensive ab initio critical point searches by providing an a priori insight into critical point structures and reaction mechanisms.

In the practical application of MM-VB, one must face the problem of identifying the *active* centers (each with one active orbital and one active electron) from the outset. The selection of active centers from the outset implies that one can only study those regions of the energy hypersurface that involve bond breaking/making between those centers. (A similar difficulty arises in MC-SCF theory itself where one must choose the active space orbitals.) In what follows, we shall consider several multi-bond reactions involving making/breaking bonds between carbon atom sites so that the choice of active centers is obvious. However, while one can describe the conformational effects of the framework via V_{MM}, clearly, one cannot allow for the bond breaking within the framework itself unless the active orbital space is enlarged.

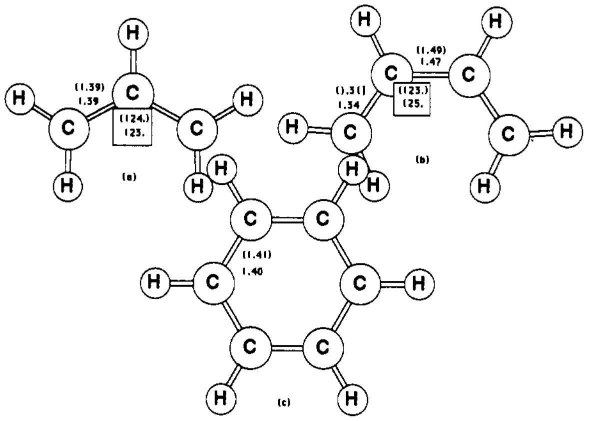


Figure 3. MM-VB and 4-31G MC-SCF (in brackets) structural parameters (Å and degrees) for some conjugated systems.

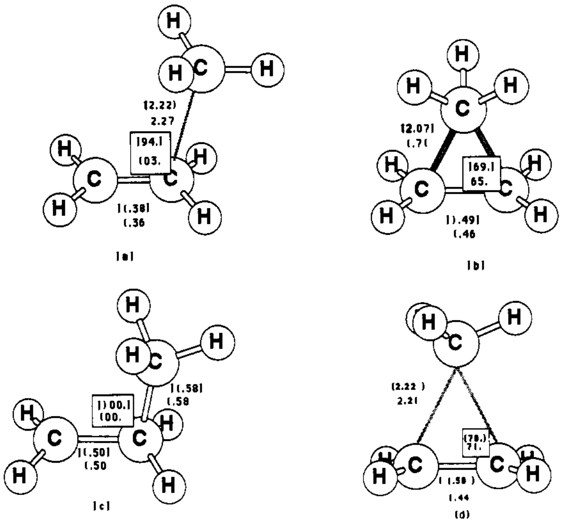


Figure 4. MM-VB and 4-31G MC-SCF (in brackets) structural parameters (Å and degrees) for the addition of $CH_3^{\bullet}$ + $CH_2{=}CH_2$: (a) transition structure for the addition reaction, (b) transition structure for the 1-2 $CH_3^{\bullet}$ shift, (c) $CH_3CH_2{=}CH_2^{\bullet}$ minimum, (d) Conical intersection between ground and excited state.

Now we shall present some data that illustrate the use of MM-VB method in practical applications. *The results have been obtained by full geometry optimization in MM-VB using the same pool of MM and VB parameters.* In Figure 3 through 7, we present scale drawings of the structures obtained from MM-VB for some selected optimized structures (minima, transition states, and conical intersections) associated with the structural chemistry of delocalized π systems (Figure 3), a simple radical addition, $CH_3^{\bullet}$ + $CH_2{=}CH_2$ (Figure 4), with a forbidden 2 + 2 cycloaddition ethylene + ethylene (Figure 5) with the simplest Diels–Alder reaction, butadiene + ethylene (Figure 6), and with the Cope rearrangement of 1,5-hexadiene (Figure 7). For the allyl radical, butadiene, and benzene the π orbitals are *active* so that we have three, four, and six active orbitals/electrons. For the multi-bond reactions we have used *one active orbital* and *one active electron per reactive carbon center.* Thus for $CH_3^{\bullet}$ + $CH_2{=}CH_2$ there are three active orbitals, one on each carbon, and three active electrons. Similarly for the 2 + 2 cycloaddition ethylene + ethylene there are four active orbitals and four active electron, while in both butadiene + ethylene and the Cope rearrangement there are six active orbitals and six active electrons. The ab initio MC-SCF results are shown in brackets. Clearly, given the increased range of generality of MM-VB as opposed to the MM method, it would be unreasonable to expect the same level of precision in computed geometrical parameters. In fact, as we shall now discuss, MM-VB performs very well.

The results from structural chemistry of delocalized π systems (allyl radical, butadiene, and benzene in Figure 3) demonstrates

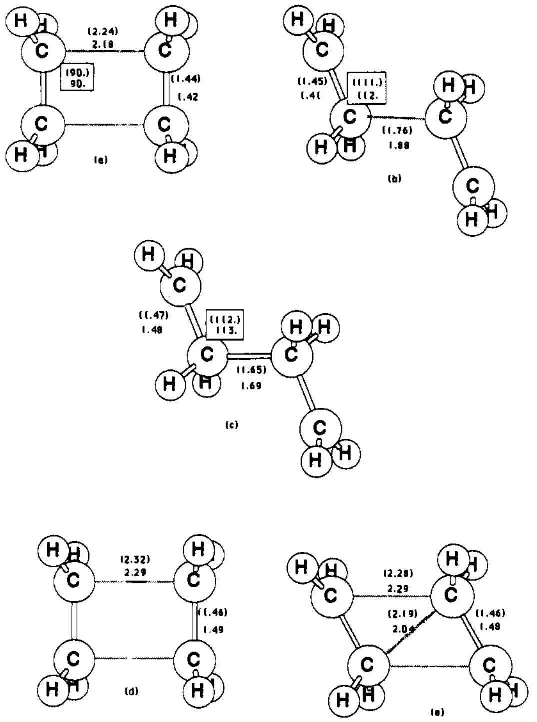


Figure 5. MM-VB and 4-31G MC-SCF (in brackets) structural parameters (Å and degrees) for the [2 + 2] cycloaddition of two ethylene molecules: (a) $2_s + 2_s$ forbidden "transition state" (a saddle point of index 2 with 2 imaginary frequencies), (b) anti biradical fragmentation transition state, (c) anti biradical minimum, (d) $2_s + 2_s$ forbidden reaction *excited* state structure (a transition state connecting 2 conical intersections[8]), (e) conical intersection geometry between ground and excited state.

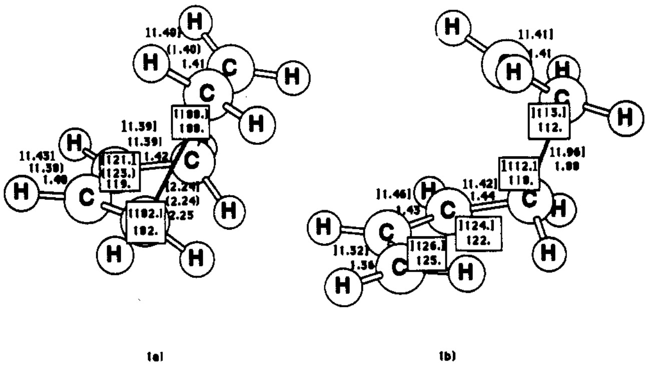


Figure 6. MM-VB and MC-SCF (6 orbital STO-3G CAS in square brackets, 4 orbital 4-31G CAS in square brackets) structural parameters for the [4 + 2] cycloaddition of ethylene and butadiene. (a) $[4_s + 2_s]$ synchronous transition state, (b) $[4_s + 2_s]$ asynchronous transition state leading to a biradical minimum.

the environmental independence of the parameters in MM-VB. It can be seen that the C–C distances are accurate to ±0.02 Å. In the butadiene molecule the two different bonds (the central bond and the two terminal bonds) are correctly described. The CCC angles are also in agreement with the ab initio results within 1.0°. In MM one requires a *different* set of parameters for each of the three types of resonance situations. The fact that MM-VB gives this accuracy with a single parameter set indicates the remarkable simplicity of the electronic structures of such systems when viewed using modern VB theory.

As we have discussed previously, our objective is to be able to describe the potential energy surface for a reactivity problem sufficiently accurately that the surface topology is correctly reproduced. This is the prerequisite condition for obtaining structural information without making any a priori assumptions about the nature of, or the existence of, any mechanistic pathway. It is convenient to discuss this aspect of our results using an example where several topological features can be illustrated on the same potential energy diagram. We shall use the $CH_3^\bullet$ + $CH_2{=}CH_2$ reaction as an example. Thus, in order to illustrate the global aspects of the surface, in Figure 8 we give the full potential energy surface (computed using MM-VB) for ground and excited states corresponding to motion of the CH_3 moiety in the plane containing the ethylene π orbitals. (In Figure 4 we give the equilibrium and transition structures as well as the geometry of the conical intersection between the ground- and excited-state surfaces.) These surfaces clearly have the correct topology (conical intersection at gridnodes 10,10 on ground and excited state, a transition state for the addition reaction at gridnodes 17,16, a transition state for the 1,2 shift at gridnodes 10,3, and the radical minimum at gridnodes 18,8). To put these results in their proper perspective one must remember that an MM approach to computing transition structures would require three different sets of parameters to generate structures for the radical minimum and the two transition states. Thus obtaining a complete potential surface would be impossible in MM. Further, the excited state surface has no minimum, only a conical intersection with the ground state. This topological feature could not be represented using an MM potential at all. Finally, we must mention, with reference to Figure 8, that a potential application of MM-VB lies in the visualization of mechanistic information. Even for this small system the generation of two-dimensional grids of energies is quite expensive using ab initio data. In contrast the MM-VB surfaces took only a few minutes to compute.

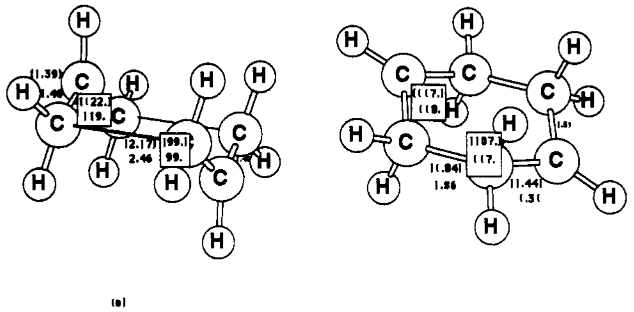


Figure 7. MM-VB and 4-31G MC-SCF (in brackets) structural parameters (Å and degrees) (a) for the concerted transition state and (b) biradical minimum in the nonconcerted Cope rearrangement of 1,5-hexadiene.

Now we shall discuss the ability of the MM-VB method to describe transition structures and reaction intermediates. In Figures 4–7 we summarize the structures obtained from computations on some selected intermediates and transition states belonging to four different potential energy surfaces where the results of detailed MC-SCF computations are available: (i) $CH_3^\bullet$ + $CH_2{=}CH_2$ (Figure 4, a–c), (ii) ethylene + ethylene (Figure 5, a–c), (iii) butadiene + ethylene (Figure 6), and (iv) the Cope rearrangement of 1,5-hexadiene (Figure 7).[11,17] We emphasize that we have used the same pool of parameters for the valence space parametrization in all computations. Note that the structure in Figure 5a (the antiaromatic structure for the 2_s+2_s forbidden cycloaddition of two ethylenes) is, in fact, a second-order saddle point with two imaginary frequencies, and this reaction cannot be studied by SCF (RHF or UHF) or semiempirical methods.

For the transition states the bond lengths agree, in general, with the ab initio results within 0.2 Å. For example, the fragmentation (from the biradical minimum, Figure 5c) transition structure (Figure 5b) for the nonconcerted 2 + 2 cycloaddition of two ethylenes (Figure 6b) has an error of 0.1 Å in the forming C–C distance. This is perhaps not too surprising since the surface is exceedingly flat with an ab initio fragmentation barrier of 0.4 kcal/mol. One exception is the symmetric transition state for the 1–2 shift in CH_3 + ethylene (Figure 4b) in which the C–C distance differs by 0.3 Å. However, this case is rather pathological because of the proximity of the conical intersection to this transition state. The CCC angles describing the orientation of the reacting fragment agree with the corresponding ab initio value within a few

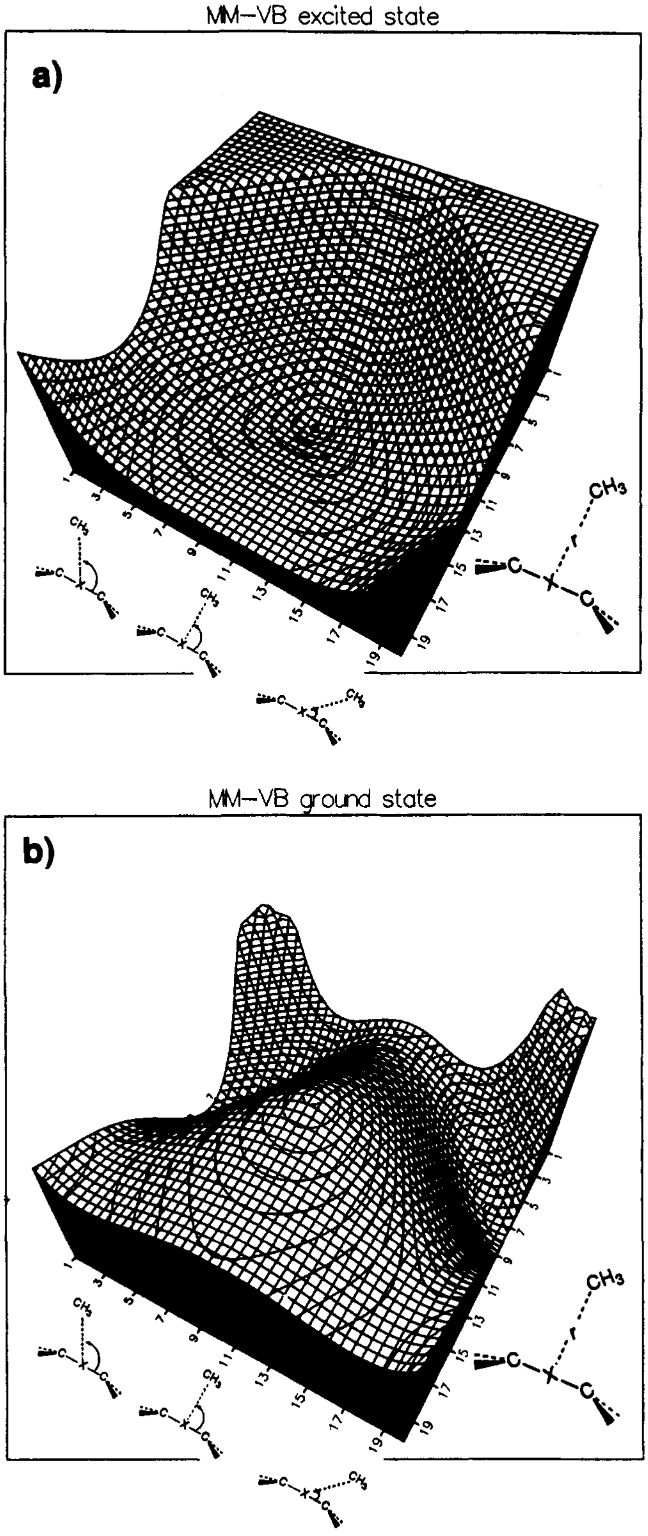


Figure 8. Potential energy surface (MM-VB) for $CH_3^{\bullet} + CH_2{=}CH_2$: (a) excited state, (b) ground state. The X and Y axes are based upon a parabolic interpolation (X axis) and linear interpolation (Y axis) passing through six structures including the optimized minima and transition states. The X axis (top right to bottom left) corresponds mainly to the distance r_{C-X} between the methyl and the midpoint of the ethylene C–C distance. Gridnode 1 corresponds to r_{C-X} = 1.49 Å and gridnode 20 corresponds to r_{C-X} = 2.68 Å. The Y axis (top left to bottom right) corresponds mainly to the angle CH_3–X–CH_2. Gridnode 1 corresponds to and angle of 127.9° and gridnode 20 to 52.1°.

degrees. The bond-length errors found in transition structures are mainly due to the crude parametrization of the delocalization effect. This problem affects the radial behavior of the K_{ab}'s and Q_{ab}'s much more than their angular behavior.

Radical and diradical intermediates normally need a set of special parameters in the MM method. In MM-VB the bond length and angle of the propyl radical H_3C-CH_2-CH_2 (Figure 4c) is in agreement with the ab initio values within 0.01 Å and 1°, respectively. Two diradical structures are illustrated: the *anti* diradical minimum that occurs in the cycloaddition reaction of two ethylenic molecules (Figure 5c) and the cyclohexane diradical minimum of the Cope rearrangement (Figure 7b). For these two structures the bond lengths are correct within 0.1 Å and the bond angles within 3.0°.

An area of intense interest that is inaccessible to MM methods concerns calculations on the excited state and crossings with ground-state potential energy surfaces. Here we present two examples that serve to illustrate that such calculations are possible. In Figure 8 one can see that MM-VB reproduces the conical intersection between the ground and excited state at gridnodes 10,10 for the $CH_3^{\bullet} + CH_2{=}CH_2$ reaction. In Figure 4b we give the structural details of this point. In Figure 5a we show that antiaromatic structure for the $2_s + 2_s$ forbidden cycloaddition of two ethylenes (which is, in fact, a second-order saddle point with two imaginary frequencies). The corresponding structure on the excited surface is shown in Figure 5b. This structure is, in fact, a saddle point connecting two equivalent conical intersections[8,15] that have the structure given in Figure 5e. As we have discussed elsewhere, conical intersections can provide the mechanism for a fully efficient return from an excited state in a photochemical reaction.[8,9a]

Of course, the utility of an MM-VB approach lies in the ability to study reactivity problems in larger molecules where ab initio computations are at the limit or beyond the range of current computing technology. In order to illustrate the type of information that can be obtained for larger systems, we have applied MM-VB to ergosterol, a steroid which is a precursor of vitamin D_2. Ergosterol undergoes an electrocyclic ring opening involving a cyclohexadiene ring to form a hexatriene system. In Figure 9 we show the structures obtained for a conformer of ergosterol (Figure 9a) and for the critical point corresponding to its ground-state conrotatory ring opening (Figure 9b). Each shaded atom has one *active* orbital. The six carbon atoms with *active* orbitals composing the cyclohexadiene system are illustrated separately. The main geometrical difference between those two structures arises from the status of the σ bond between the active atoms A and B. The A–B bond, which is in its equilibrium geometry in the first structure, is undergoing bond breaking in the second. We can now observe that MM-VB not only describes the change in the hybridization status of the two centers A and B but also that MM-VB provides a certain amount of insight into the conformational changes occurring in the nonreactive part of the molecule. In particular, one can see that the two cyclohexane ring systems which are condensed with the active cycloexadiene ring show a different conformation in the two cases. They are in a clear *chair* conformation in the stable structure of Figure 9a but a quasi-*half-chair* conformation in the transient structure of Figure 9b. As a consequence of these conformational differences, a more macroscopic effect is induced on the orientation of the long side hydrocarbon chain with respect to the plane of the active cyclohexadiene ring. The driving force behind these conformational changes has its origin in the change of hybridization status of carbon atoms A and B. This change in hybridization status arises, in turn, from spin-recoupling that arises during bond breaking and is described by the quantum mechanics of the VB problem. Thus with MM-VB one can explore the coupling between the center of the reaction (which is described by the VB simulation) and the description of more distant regions of the same molecule which undergoes conformational changes (which are described via MM). Of course, the most interesting part of

---

(15) Bernardi, F.; Bottoni, A.; Robb, M. A.; Schlegel, H. B.; Tonachini, G. *J. Am. Chem. Soc.* **1985**, *107*, 2260–2264.

(16) Bernardi, F.; Bottoni, A.; Field, M. J.; Guest, M. F.; Hillier, I. H.; Robb, M. A.; Venturini, A. *J. Am. Chem. Soc.* **1988**, *110*, 3050–3055.

(17) Morokuma, K.; Borden, W. T.; Hrovat, D. A. *J. Am. Chem. Soc.* **1988**, *110*, 4474.

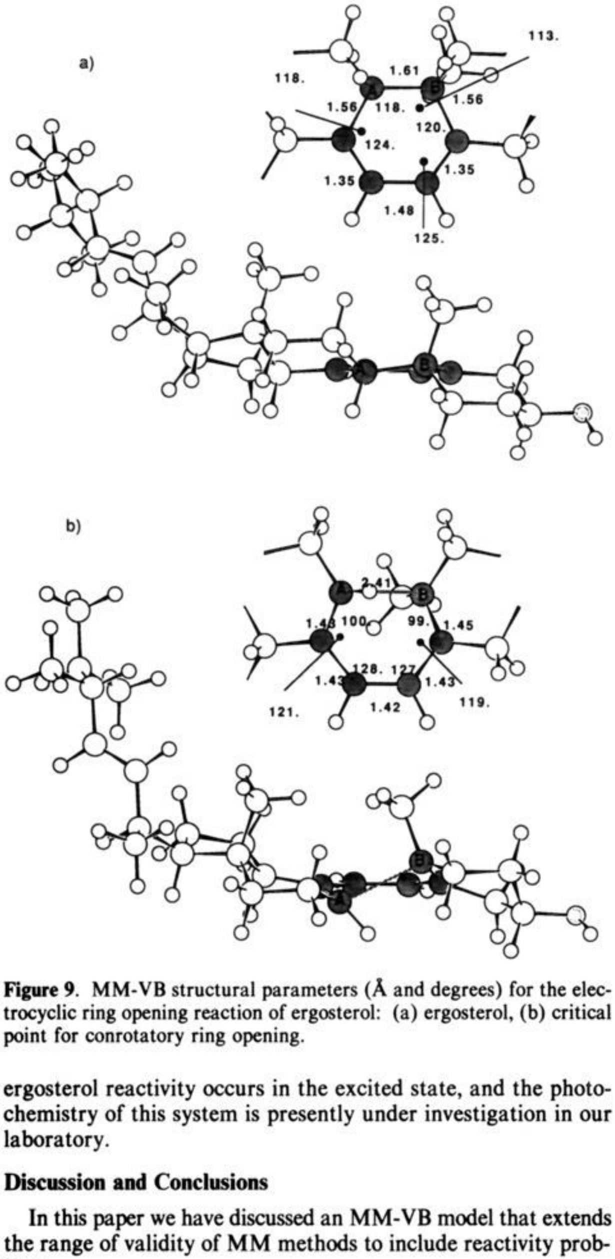


Figure 9. MM-VB structural parameters (Å and degrees) for the electrocyclic ring opening reaction of ergosterol: (a) ergosterol, (b) critical point for conrotatory ring opening.

ergosterol reactivity occurs in the excited state, and the photochemistry of this system is presently under investigation in our laboratory.

Discussion and Conclusions

In this paper we have discussed an MM-VB model that extends the range of validity of MM methods to include reactivity problems.

The MM force field is dependent on a local environment defined by specifying different atom types as input to the procedure. As a consequence, the parameters describing a double bond in an ethylene are not the same as those for describing a double bond in a butadiene or other conjugated hydrocarbon. Similarly, to describe a transition state one needs special atom types and associated parameters. In contrast, in MM-VB, we have defined a new atom type that has the *active orbitals* that are used in a VB computation. The concept of an *active orbital space* is associated with the reactive process under investigation and comprises the orbitals that are associated with bond making/breaking. A parametrized pool of K_{ab}'s and Q_{ab}'s is associated with the active orbital space and allows a smooth description of the bond-breaking and bond-forming processes through the solution of a simple VB problem. Consequently, one has an environment-independent description of various bond-making/breaking situations that is impossible in MM. Thus, for example, the *complete* potential surface corresponding to the breaking of π bonds and making of new σ bonds that occurs in, say, a cycloaddition reaction can be described with the *same* pool of parameters.

We have thus demonstrated that the field of application of MM-VB is much larger than for pure MM. In MM-VB, chemical reactivity problems in ground and excited states can be studied. The main limitation arises from the problem of identifying the *active* centers from the outset. (A similar difficulty arises in MC-SCF theory itself where one must choose the active space orbitals.) This problem is obviously much less severe than the problem of identifying an atom type (and associated hybridization status) in MM. However, the selection of active centers from the outset implies that one can only study those regions of the energy hypersurface that involve bond breaking/making between those centers.

We have illustrated the limitations and capabilities of MM-VB in computing the geometries of intermediates and transition states using results obtained for some critical points belonging to four different potential energy surfaces. The results have been obtained using the same pool of parameters for the valence space parametrization in all computations. In the examples we have studied, the correct shape (topology) of the potential energy surfaces is predicted. An ab initio MC-SCF geometry optimization starting from parameters computed from MM-VB usually converges rapidly.

A fundamental limitation of the MM method is associated with the calculation of geometries and transition structures for electronic excited states of a molecule. This would be possible in the MM method only by extending the pool of parameters to an even larger set for each excited state and spin multiplicity. In the MM-VB method, covalent excited states (irrespective of spin multiplicity) can be treated with the same pool of parameters as the ground states. For example, we have located the excited-state structure that is located at approximately the same geometry as the antiaromatic ground-state critical point on the supra–supra approach in the ethylene + ethylene reaction. For this structure the MM-VB bond lengths differ from the ab initio value by only 0.03 Å, while the angles are exactly equal by symmetry. Further, the conical intersection that must occur between the ground and the excited states of the potential energy surfaces of $CH_3^{\bullet} + CH_2{=}CH_2$ and of the cycloaddition of two ethylenes are correctly reproduced. Thus the delicate interplay between ground- and excited-state surfaces that is necessary to understand the photochemistry is correctly reproduced.

There are two important general conclusions. On the one hand, we have demonstrated that MM-VB could evolve to become a useful practical tool for the rapid investigation of reactivity problems before much more expensive ab initio computations are carried out. The main limitation to the accuracy in the present model is the model for the delocalization effect. On the other hand, the fact that the MM-VB modelling algorithm works at all is of considerable conceptual importance, and we must now briefly comment on this point. We have demonstrated that quantum mechanics of the complicated process of the reorganization of the spin recoupling that occurs in a transition state or the driving force for the change in CC bond lengths that occurs in delocalized π systems can both be described by the solution of a simple VB problem which allows for the *resonance* of the possible VB structures *involving the active orbitals alone*. The inactive *framework* of the molecule can be described by MM. The central parameters in such a VB calculation are the exchange integrals (K_{ab}) that depend mainly on the overlap of the orbitals. The functional form of these exchange parameters can be transferred from model molecules (in the present case ethane and ethylene) using a delocalization algorithm.

Appendix: Formulas for MM-VB

In this Appendix we give the formulas that are used in addition to the standard MM2 potentials.

(i) Two-Orbital Model Q^0_{ij} and K^0_{ij}. The model Q^0_{ij} and K^0_{ij} for ethane and ethylene have been fitted to a functional form given in eq A1 and A2. The coefficients c_{is}, c_{ipx}, c_{ipy}, c_{ipz}, and c_{js}, c_{jpx},

$$K^0_{ij} = c_{ip_x}c_{jp_x}\mathrm{Ex}_1(ij) + (c_{ip_y}c_{jp_y} + c_{ip_z}c_{jp_z})\mathrm{Ex}_2(ij) + (c_{ip_x}c_{js} + c_{is}c_{jp_x})\mathrm{Ex}_3(ij) + c_{is}c_{js}\mathrm{Ex}_4(ij) \quad \text{(A1)}$$

$$Q^0{}_{ij} = (c^2_{ip_x} + c^2_{jp_x})\mathrm{Ex}_5(ij) + \left(c^2_{ip_y} + c^2_{jp_y} + c^2_{ip_z} + c^2_{jp_z}\right)\mathrm{Ex}_6(ij) + (c^2_{is} + c^2_{js})\mathrm{Ex}_7(ij) + (c^2_{ip_x}c^2_{jp_x})\mathrm{Ex}_8(ij) + \left(c^2_{ip_y}c^2_{jp_y} + c^2_{ip_z}c^2_{jp_z}\right)\mathrm{Ex}_9(ij) + (c^2_{is}c^2_{js})\mathrm{Ex}_{10}(ij) + (c^2_{ip_x}c^2_{js} + c^2_{is}c^2_{jp_x})\mathrm{Ex}_{11}(ij) + \left(c^2_{ip_y}c^2_{js} + c^2_{is}c^2_{jp_y} + c^2_{ip_z}c^2_{js} + c^2_{is}c^2_{jp_z}\right)\mathrm{Ex}_{12}(ij) + \left(c^2_{ip_y}c^2_{jp_x} + c^2_{ip_x}c^2_{jp_y} + c^2_{ip_z}c^2_{jp_x} + c^2_{ip_x}c^2_{jp_z}\right)\mathrm{Ex}_{13}(ij) + \left(c^2_{ip_z}c^2_{jp_y} + c^2_{ip_y}c^2_{jp_z}\right)\mathrm{Ex}_{14}(ij) + \mathrm{Ex}_{15}(i) + \mathrm{Ex}_{15}(j) \quad \text{(A2)}$$

c_{jpy}, c_{jpz} depend only on the geometry and are not free parameters. For each $k = 1, 14$ the function $\mathrm{Ex}_k(ij)$ has the form

$$\mathrm{Ex}_k(r_{ij}) = a_k \exp(-b_k r_{ij}) \quad \text{(A3)}$$

and the parameters a_k and b_k are given in Table I with r_{ij} being the distance between the centers i and j. The units of r_{ij} are Å and the units of Ex_i are E_h. Ex_{15} depends only on i or j and is a function of the coefficient (c_{is}):

$$\mathrm{Ex}_{15}(c_{is}) = a_{15} \exp(-b_{15}\{c_{is}[\tan^{-1}(400c_{is})/\tan^{-1}(400)]\}) \quad \text{(A4)}$$

The quantities c_{is}, c_{ipx}, c_{ipy}, c_{ipz}, and c_{js}, c_{jpx}, c_{jpy}, c_{jpz} depend only on the geometry of the centers i and j and we now discuss their definition. We begin by defining a vector p_i as shown in Scheme I. The direction of the vector p_i is taken orthogonal to the plane α containing three substituents (vectors a, b, and c, assumed to be of unit length) connected to the valence center. The coefficient c_{is} is the norm of vector p_i and is proportional to the distance q between the valence atom and the plane α.

The quantities c_{ipx}, c_{ipy}, c_{ipz}, and c_{jpx}, c_{jpy}, c_{jpz} are the projections of unit vectors directed along p_i and p_j on two parallel local coordinate systems defined in Scheme II. Since Ex_k contains two parameters (a_k and b_k), a maximum of 30 different parameters are needed for any two-center system.

(ii) Delocalization Effect (11 Parameters: *e*, *f*, *g*, *h*, s_1, s_2, s_3, t_1, t_2, t_3, and *u*). The π (ethylenic) and σ (from ethane) $K^0{}_{ij}$ and $Q^0{}_{ij}$ are corrected using different algorithms. We begin with the π contribution (i.e., the correction to K_{ij} or Q_{ij} of the model π system)

$$K_{ij} = K^0{}_{ij} + \Omega\Delta K_{ij} \quad \text{(A5)}$$

where ΔK_{ij} is the change in the model K_{ij} and is defined as

$$\Delta K_{ij} = h\Lambda_{ij}\frac{(A_i + A_j)}{2} \quad \text{(A6)}$$

with

$$\Lambda_{ij} = \left[g\frac{(A_i - A_j)^2}{(A_i + A_j)^2} - 1\right] \quad \text{(A7)}$$

and

$$A_i = \sum_x K^0{}_{ix} \quad \text{(A8)}$$

where the summation over x is over all unconnected (to i) valence centers. A valence center x is unconnected to the valence center i when no permanent (MM) bond binds j to i. The function Ω only depends on the geometry of the π bonds and contains no parameters.

$$\Omega = \prod_{kl}(1 - \cos(\chi_{kl})) \quad \text{(A9)}$$

The sum over kl is over all π bonds and χ_{kl} is a "twisting angle" and is defined as the dihedral angle formed by the p_k and p_l vectors along a π bond. Finally

$$Q_{ij} = Q^0{}_{ij} - \bar{\Omega}\Delta K_{ij} \quad \text{(A10)}$$

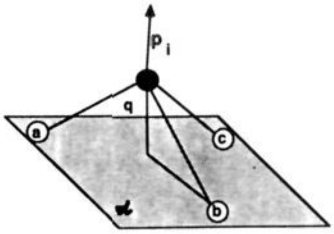
with

$$\bar{\Omega} = \sum_{kl}(1 - \cos(\chi_{kl})) \quad \text{(A11)}$$

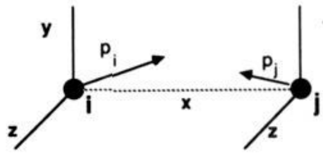
Table I. Parameters in Eq A3

Ex_i	σ bond		π bond	
i	a_i	b_i	a_i	b_i
1	−10.5730	2.4277	−39.0149	1.6548
2	−16.1738	3.2712	−3.3012	2.7429
3	−2.1898	1.6498	−23.2383	1.5080
4	−0.2388	1.9879	−9.9046	1.2424
5	−10.8428	1.3178	0.0000	0.0000
6	−0.0968	0.0009	0.0000	0.0000
7	−0.0911	0.0000	−8 306.5530	5.4211
8	73.6261	3.6707	0.0000	0.0000
9	0.0031	0.0000	16 898.8360	5.4368
10	7.5542	1.2540	0.0000	0.0000
11	119.1520	2.0906	4.0861	0.3951
12	5.2357	1.9516	0.0000	0.0000
13	58.2474	2.8313	16 662.2340	5.4240
14	81.0041	2.5974	16 547.6600	5.4154
15	0.0002	11.7333	0.0002	11.7333

Scheme I



Scheme II



The free parameters are g and h and have the values 1.9 and 0.2 in all our computations. The σ-bond model $K^0{}_{ij}$ is corrected according to

$$K_{ij} = e\Xi_\pi K^0{}_{ij}\left(\Omega + \frac{(1 - \Omega)}{e\Xi_\pi}\right) + \Omega(1 - e\Xi_\pi)\left(\sum_x \frac{K^0{}_{ix}}{2} + \sum_y \frac{K^0{}_{jy}}{2}\right) \quad \text{(A12)}$$

with

$$\Xi_\pi = 1 + \mathrm{f}(1 + ((\sum_{x>y}\Lambda_{xy})/\sum_{x>y}1)) \quad \text{(A13)}$$

and the summations over x and y are over valence atoms connected (via MM).

The corresponding correction to $Q^0{}_{ij}$ for a model σ-bond subsystem is given as

$$Q_{ij} = Q^0{}_{ij} - \bar{\Omega}\lambda_1 \exp(-\lambda_2(K^0{}_{ij} - \lambda_3)^2) + \bar{\Omega}\lambda_1 \exp(-\lambda_2(-\lambda_3)^2) \quad \text{(A14)}$$

where:

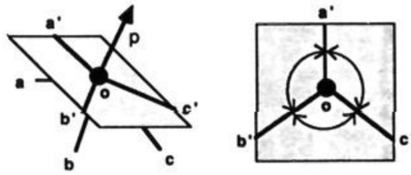
$$\lambda_1 = (t_1 - s_1)\left(\frac{\Xi_\pi - 1}{u}\right) + s_1 \quad \text{(A15)}$$

$$\lambda_2 = (t_2 - s_2)\left(\frac{\Xi_\pi - 1}{u}\right) + s_2 \quad \text{(A16)}$$

$$\lambda_3 = (t_3 - s_3)\left(\frac{\Xi_\pi - 1}{u}\right) + s_3 \quad \text{(A17)}$$

The parameters e, f, s_1, s_2, s_3, t_1, t_2, t_3, and u are free parameters with values 0.780, 0.065, 0.2, 7.0, −0.41, 0.27, 7.0, −0.34, and 0.1079.

Scheme III



(iii) Corrections to MM. The molecular mechanics parameters for stretching, bending, torsions and van der Waals interactions involving one or more valence centers have in general been set equal to those of a sp^3 carbon atom. Exception is made only when the following situations are encountered.

(1) When a bond or planar angle includes at least one valence σ bond or a dihedral angle includes at least two valence σ bonds, their respective stretching, bending, or torsional constants are set equal to zero.

(2) When the anchor atom of a planar angle is a valence center, the value of the angle is replaced by the value of the *projected angle*. The MM parameters are not changed, only the definition of the angle. This *projected angle* is defined according to the Scheme III as the angle a′ob′ or b′oc′ or c′oa′ between the projection of the bonds (ao, bo, and co) onto the plane orthogonal to the vector p defined in Scheme I.

(3) When a dihedral angle includes one valence σ bond, the torsional constants are multiplied by a term Θ_{ij} whose value in turn depends on the value of the function $\Gamma_{ij} = c_{ipx}c_{jpx}\mathrm{Ex}_1(ij)$ (a function of the bond length r_{ij} and the orientation of p_i and p_j defined in Scheme AI). The form of Θ_{ij} is:

$$\Theta_{ij} = 0.85\left\{\left(1+\frac{\tan^{-1}(4.0(2.0\Gamma_{ij}-\tfrac{1}{2}))}{\tan^{-1}(4.0)}\right)-\left(1+\frac{\tan^{-1}(2.0)}{\tan^{-1}(4.0)}\right)\right\} \tag{A18}$$

Acknowledgment. This work was supported by the SERC (UK) under Grant No. GR/F 48029, 46452, and GR/G 03335. All ab initio computations were carried out using MC-SCF programs[18] that are available in GAUSSIAN 90.[19]

Registry No. Ergosterol, 57-87-4.

---

(18) (a) Eade, R. H. A.; Robb, M. A. *Chem. Phys. Lett.* **1981**, *83*, 362. (b) Schlegel, H. B.; Robb, M. A. *Chem. Phys. Lett.* **1982**, *93*, 43.

(19) GAUSSIAN 90: Frisch, M. J.; Head-Gordon, M.; Trucks, G. W.; Foresman, J. B.; Schlegel, H. B.; Raghavachari, K.; Robb, M.; Binkley, J. S.; Gonzalez, C.; Defrees, D. J.; Fox, D. J.; Whiteside, R. A.; Seeger, R. A.; Melius, C. F.; Baker, J.; Martin, R. L.; Kahn, L. R.; Stewart, J. J. P.; Topiol, S.; Pople, J. A. Gaussian Inc.: Pittsburgh, PA.

The Nonplanar Amide Group in *N*-Acylaziridines: Conformational Behavior and Chiroptical Properties

G. V. Shustov,[*,†] **G. K. Kadorkina,**[†] **S. V. Varlamov,**[†] **A. V. Kachanov,**[†]
R. G. Kostyanovsky,[†] **and A. Rauk**[*,‡]

Contribution from the Institute of Chemical Physics, Academy of Sciences of the U.S.S.R., 117977 Moscow, U.S.S.R., and Department of Chemistry, The University of Calgary, Calgary, Canada T2N 1N4. Received June 10, 1991

Abstract: Chiroptical properties in the intrinsically dissymmetric amide chromophore of *N*-acylaziridines are investigated experimentally and by ab initio molecular orbital calculations. Computations on *N*-formylaziridine (**1a**) and measurements of the CD spectra of (1*R*,2*R*)-1-acetyl-2-methylaziridine (**1b**), (1*R*,2*R*)-1-(α-hydrohexafluoroisobutyryl)-2-methylaziridine (**1c**), (2′*S*)-1-(2′-methylbutyryl)aziridine (**1d**), and (2′*S*)-1-(2′-methylbutyryl)-2,2-dimethylaziridine (**1e**) suggest that the Cotton effect (CE) for the long wavelength absorption (band I) obeys a spiral rule: a right-handed twist of the O=C—N—C(ring) bonds corresponds to a negative CE for band I. Band I is assigned to the valence n_O–π^*_{CO} transition. The next four higher energy absorptions are due to transitions to Rydberg states, n_N–3s and n_N–3p. Experimental CD spectra are also reported for the related compounds, (2′*S*)-1,2-bis(2′-methylbutyryl)-3,3-dimethyldiaziridine (**2a**), (α*S*)-1,2-diacetyl-3-*sec*-butyldiaziridine (**2b**), (2′*S*)-2-(2′-methylbutyryl)-3,3-dimethyloxaziridine (**3a**), (5*S*)-2-acetyl-5-methyl-1-oxa-2-azaspiro[2.5]octane (**3b**), (*S*)-*N*,*N*-dimethyl-2-methylbutyramide (**4**), and (2*S*)-1-(α-hydrohexafluoroisobutyryl)-2-methylazetidine (**5**).

Introduction

N-Acyl-substituted three-membered nitrogen heterocycles present a certain interest as compounds containing the most nonplanar amide group. The high degree of nonplanarity of such amides follows from the results of structural studies in the solid[1] and gaseous[2] phases, and is confirmed by the data of IR, ^{1}H NMR, and UV spectra in solution.[3] Nonplanarity and, connected with it, inherent chirality of the amide chromophore in *N*-acylaziridines strongly affect the circular dichroism (CD) spectra,[4] for the interpretation of which a correct assignment of optically active electronic transitions is required. Hitherto, assignments for the nonplanar amide chromophore are based only upon semiempirical calculations.[5] The experimental studies of the chiroptical

---

[*] Author to whom correspondence should be addressed.

[†] Academy of Sciences of the U.S.S.R.

[‡] The University of Calgary.

(1) (a) Shibaeva, R. P.; Atovmyan, L. O.; Kostyanovsky, R. G. *Dokl. Akad. Nauk SSSR* **1967**, *175*, 586–589. (b) Dyachenko, O. A.; Atovmyan, L. O.; Aldoshin, S. M.; Polyakov, A. E.; Kostyanovsky, R. G. *J. Chem. Soc., Chem. Commun.* **1976**, 50.

(2) (a) Vilkov, L. V.; Nazarenko, I. I.; Kostyanovsky, R. G. *Zh. Strukt. Khim.* **1968**, *9*, 1075–1077. (b) Tarasenko, N. A.; Avakyan, V. G.; Belik, A. V. *Ibid.* **1978**, *19*, 541–543.

(3) (a) Kostyanovksy, R. G.; Bystrov, V. F. *Dokl. Akad. Nauk SSSR* **1963**, *148*, 839–842. (b) Anet, F. A. L.; Osyany, J. M. *J. Am. Chem. Soc.* **1967**, *89*, 352–356. (c) Kodera, Y.; Kogama, M.; Mori, F.; Ohta, H. *Fukuoka Daigaku Rigaku Shuno* **1978**, *8*, 11–15; *Chem. Abstr.* **1980**, *93*, 185614q. (d) Papoyan, T. Z.; Chervin, I. I.; Kostyanovsky, R. G. *Izv. Akad. Nauk SSSR, Ser. Khim.* **1978**, 1530–1543. (e) Kostyanovsky, R. G.; Zakharov, K. S.; Zaripova, M.; Rudtchenko, V. F. *Tetrahedron Lett.* **1974**, 4207–4210.

0002-7863/92/1514-1616$03.00/0 © 1992 American Chemical Society